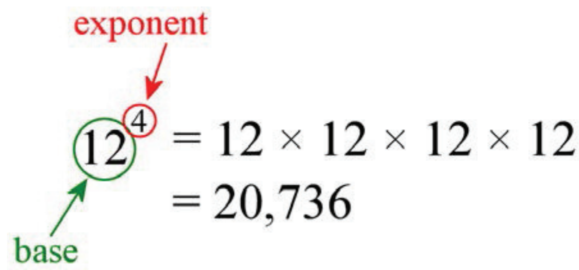
Powers and Exponents

Exponents are a “shorthand” for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as “two to the fifth power,” “two to the fifth,” or “two raised to the fifth power.”

Similarly, 7^9 is read as “seven to the ninth power,” “seven to the ninth,” or “seven raised to the ninth power.”

The “powers of 6” are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something “**squared.**” For example, 11^2 is read as “**eleven squared.**” That is because it gives us the area of a square with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word “**cubed.**” For example, 31^3 is read as “**thirty-one cubed**” because it gives the volume of a cube with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them using mental math.

a. $3^2 = \underline{3 \times 3 = 9}$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

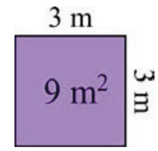
c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

You just learned that the expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with a side length of 7 units. Let's compare that to square meters and other units of area.



If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square meters.

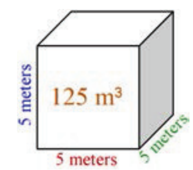
Notice that the symbol for square meters is $\mathbf{m^2}$. This means "**meter × meter**." We are, in effect, squaring the unit *meter* (multiplying the unit of length *meter* by itself)!

Or, in the expression 9 cm × 9 cm, we multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 $\mathbf{cm^2}$**, and the square centimeter (cm^2) comes from multiplying "**centimeter × centimeter**."

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometers or square millimeters.

With the customary units of area, such as square inches, square feet, and square miles, people often write "sq. in.", "sq. ft.", or "sq. mi." , instead of $\mathbf{in^2}$, $\mathbf{ft^2}$, and $\mathbf{mi^2}$. Both ways are correct.

In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *meter* by itself three times (meter × meter × meter) to get the unit of volume "cubic meter" or m^3.



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometers: A = 12 km × 12 km = ________________	**b.** A square with sides 6 m long: A = ________________
c. A square with a side length of 6 inches: A = ________________	**d.** A square with a side with a length of 12 ft: A = ________________

4. Express the volume (V) as a multiplication, and solve.

a. A cube with a side of 2 cm: V = 2 cm × 2 cm × 2 cm = ________	**b.** A cube with sides each 10 inches long: V = ________________
c. A cube with sides 1 ft in length: V = ________________	**d.** A cube with edges that are all 5 m long: V = ________________

5. **a.** The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic inches. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is its edge length?

The powers of 10 are very special —and very easy!

Notice that the exponent tells us *how many zeros* there are in the answer.

$10^1 = 10$	$10^4 = 10,000$
$10^2 = 10 \times 10 = 100$	$10^5 = 100,000$
$10^3 = 10 \times 10 \times 10 = 1,000$	$10^6 = 1,000,000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2,187$, how can you use that result to find 3^8 ?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35,184,372,088,832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the _____________ of a _______________ with a side length of ______ units.

b. 101^3 gives us the _____________ of a _______________ with an edge length of ______ units.

c. 2×6^2 gives us the ____________ of two _______________ with a side length of ______ units.

d. 4×10^3 gives us the ____________ of ____ _______________ with an edge length of ______ units.

Puzzle Corner

Make a pattern, called a *sequence*, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, n. What value do you calculate for n^0?
Do you think it will come out this way for every base number?
Why or why not?

The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but do not worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum ($b + c$) so that you multiply the numbers b and c separately by a, and lastly, add.

You have already used it! For example, think of $3 \cdot 84$ as $3 \cdot (80 + 4)$. You can then multiply 80 and 4 *separately by 3*, and lastly, add: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$. We called this the partial products or multiplying in parts.

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply <u>**both**</u> 3 and t by s:

$s \cdot (3 + t) = s \cdot 3 + s \cdot t$, which simplifies to $3s + st$.

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot$ ___ $+ 3 \cdot$ ___ $=$	**b.** $7(50 + 6) = 7 \cdot$ ___ $+ 7 \cdot$ ___ $=$
c. $4(a + b) = 4 \cdot$ ___ $+ 4 \cdot$ ___ $=$	**d.** $2(x + 6) = 2 \cdot$ ___ $+ 2 \cdot$ ___ $=$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply **each term** in the sum by the factor in front of the parentheses:

$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6$, which simplifies to $5x + 5y + 30$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

Example 4. Now, one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

3. Multiply using the distributive property.

a. $2(3x + 5) =$	**b.** $7(7a + 6) =$
c. $5(4a + 8b) =$	**d.** $2(4x + 3y) =$
e. $3(9 + 10z) =$	**f.** $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	**h.** $8(5 + 2a + 3b) =$

To understand even better why the the distributive property works, let's look at an area model (this, too, you have seen before!).

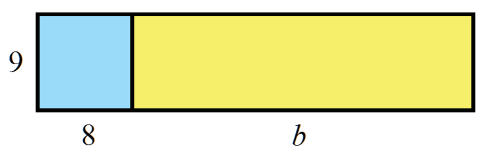
The area of the whole rectangle is 5 times (b + 12).

But, if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.

Of course, these two expressions have to be equal:

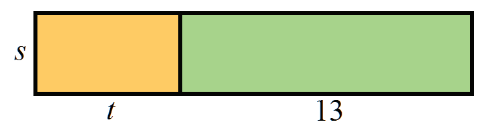
$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$

4. Write an expression for the area in two ways, thinking of one rectangle or two.



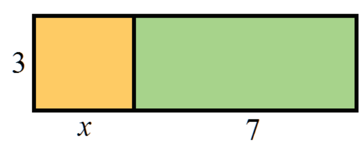
a. 9(_____ + _____) and

9 · _____ + 9 · _____ =

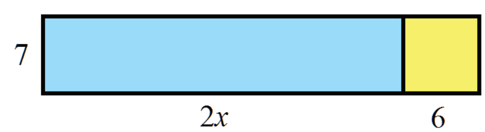


b. s(_____ + _____) and

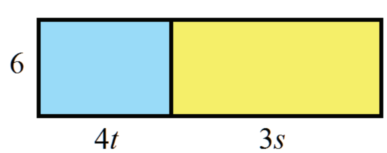
s · _____ + s · _____ =



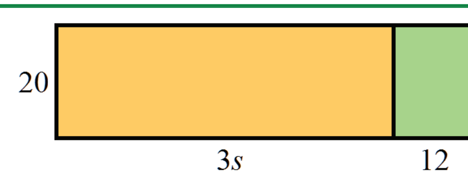
c. ____ (_____ + _____) and



d.

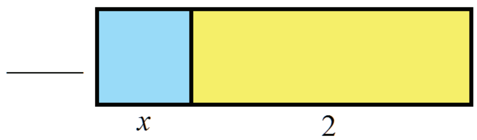


e.



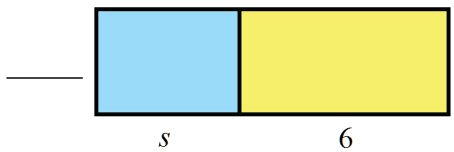
f.

5. Find the missing number or variable in these area models.



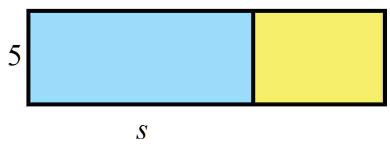
a. ____ $(x + 2) = 3x + 6$

b. _____ $(t + 8) = 7t + 56$

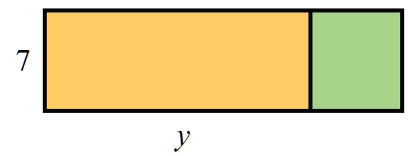


c. The total area is $9s + 54$.

d. 4(____ + 5) $= 4z + 20$



e. $5(s +$ ____ $) = 5s + 30$



f. The total area is $7y + 42$.

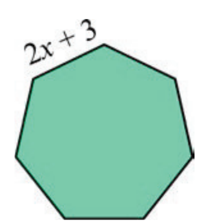
6. Find the missing number in the equations.

a. ____ $(x + 5) = 6x + 30$	**b.** $10(y +$ ____ $) = 10y + 30$
c. 6(____ $+ z) = 12 + 6z$	**d.** $8(r +$ ____ $) = 8r + 24$

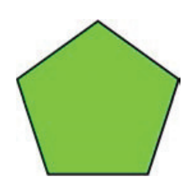
7. Find the missing number in the equations. These are just a little bit trickier!

a. ____ $(2x + 5) = 6x + 15$	**b.** ____ $(3w + 5) = 21w + 35$
c. ____ $(6y + 4) = 12y + 8$	**d.** ____ $(10s + 3) = 50s + 15$
e. 2(_____ + 9) $= 4x + 18$	**f.** 4(______ + 3) $= 12x + 12$
g. 5(_____ + 3) $= 20y + 15$	**h.** 8(____ + ____ + 7) $= 40t + 8s + 56$

8. Write an expression for the perimeter of this regular heptagon, as a <u>product</u>.
Then, multiply the expression using the distributive property



9. The perimeter of a regular pentagon is $15x + 5$. How long is one of its sides?



When we use the distributive property "backwards", and write a sum as a product, it is called **factoring**.
Example 5. The sum $5x + 5$ can be written as $5(x + 1)$. We took the SUM $5x + 5$ and wrote it as a PRODUCT— something times something, in this case 5 times the quantity $(x + 1)$.
Example 6. The sum $24x + 16$ can be written as the product $8(3x + 2)$.   *Notice* that the numbers 24 and 16 are *both divisible by 8*! That is why we write 8 as one of the factors.

10. Think of the distributive property "backwards", and factor these sums. Think of divisibility!

a. $6x + 6 =$ ____ $(x + 1)$	**b.** $8y + 16 = 8($ ____ $+$ ____ $)$
c. $15x + 45 =$ ____ $(x +$ ____ $)$	**d.** $4w + 40 =$ ____ $(w +$ ____ $)$
e. $6x + 30 =$ ____ (____ + ____)	**f.** $8x + 16y + 48 =$ ____ (____ + ____ + ____)

11. Factor these sums (writing them as products). Think of divisibility!

a. $8x + 4 =$ ____ $(2x +$ ____ $)$	**b.** $15x + 10 =$ ____ $(3x +$ ____ $)$
c. $24y + 8 =$ ____ (____ + ____)	**d.** $6x + 3 =$ ____ (____ + ____)
e. $42y + 14 =$ ____ (____ + ____)	**f.** $32x + 24 =$ ____ (____ + ____)
g. $27y + 9 =$ ____ (____ + ____)	**h.** $55x + 22 =$ ____ (____ + ____)
i. $36y + 12 =$ ____ (____ + ____)	**j.** $36x + 9z + 27 =$ ____ (____ + ____ + ____)

12. The perimeter of a square is $48x + 16$. How long is its side?

Puzzle Corner

As a storekeeper, you need to purchase 1,000 items to get a wholesale (cheaper) price of \$8 per item, so you do. You figure you might sell 600 of them. You also want to advertise a \$3 discount to your customers. What should the non-discounted selling price be for you to actually earn a \$500 profit from the sale of these items?

Epilogue: It may be hard to see now where distributive property or factoring might be useful, but it IS extremely necessary later in algebra, when solving equations.

To solve the problem above, you *can* figure it out without algebra, but it becomes fairly straightforward if we write an equation for it. Let p be the non-discounted price. We get

$$600(p - \$3) = 1{,}000 \cdot \$8 + \$500$$

To solve this equation, one needs to use the distributive property in the very first step:

$$600p - \$1800 = \$8{,}500$$

$$600p = \$10{,}300$$

(Can you solve this last step yourself?)