

# LESSON 3.2 Rate of Change and Slope

 **FL CC 8.F.2.4**

... Determine the rate of change... of the function from... two  $(x, y)$  values, including reading these from a table or from a graph....



## ESSENTIAL QUESTION

How do you find a rate of change or a slope?

## Investigating Rates of Change

A **rate of change** is a ratio of the amount of change in the output to the amount of change in the input.

### EXAMPLE 1



 **FL CC 8.F.2.4**

Eve keeps a record of the number of lawns she has mowed and the money she has earned. Tell whether the rates of change are constant or variable.

	Day 1	Day 2	Day 3	Day 4
<b>Number of lawns</b>	1	3	6	8
<b>Amount earned (\$)</b>	15	45	90	120

**STEP 1** Identify the input and output variables.

Input variable: number of lawns    Output variable: amount earned

**STEP 2** Find the rates of change.

$$\text{Day 1 to Day 2: } \frac{\text{change in \$}}{\text{change in lawns}} = \frac{45 - 15}{3 - 1} = \frac{30}{2} = 15$$

$$\text{Day 2 to Day 3: } \frac{\text{change in \$}}{\text{change in lawns}} = \frac{90 - 45}{6 - 3} = \frac{45}{3} = 15$$

$$\text{Day 3 to Day 4: } \frac{\text{change in \$}}{\text{change in lawns}} = \frac{120 - 90}{8 - 6} = \frac{30}{2} = 15$$

The rates of change are constant: \$15 per lawn.



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### Math Talk

**Mathematical Practices**

Would you expect the rates of change of a car's speed during a drive through a city to be constant or variable? Explain.

### YOUR TURN

- The table shows the approximate height of a football after it is kicked. Tell whether the rates of change are constant or variable.

Find the rates of change:

\_\_\_\_\_

The rates of change are **constant / variable.**

Time (s)	Height (ft)
0	0
0.5	18
1.5	31
2	26



**Personal Math Trainer**

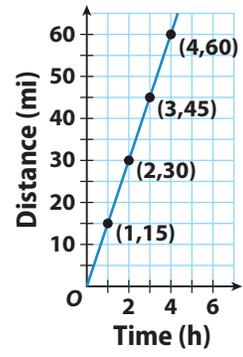
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# Using Graphs to Find Rates of Change

You can also use a graph to find rates of change.

The graph shows the distance Nathan bicycled over time.  
What is Nathan's rate of change?



**A** Find the rate of change from 1 hour to 2 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{30 - \boxed{\phantom{00}}}{2 - 1} = \frac{\boxed{\phantom{00}}}{1} = \boxed{\phantom{00}} \text{ miles per hour}$$

**B** Find the rate of change from 1 hour to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \boxed{\phantom{00}}}{4 - \boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \boxed{\phantom{00}} \text{ miles per hour}$$

**C** Find the rate of change from 2 hour to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \boxed{\phantom{00}}}{4 - \boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \boxed{\phantom{00}} \text{ miles per hour}$$

**D** Recall that the graph of a proportional relationship is a line through the origin. Explain whether the relationship between Nathan's time and distance is a proportional relationship.

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## Reflect

**2. Make a Conjecture** Does a proportional relationship have a constant rate of change?

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**3.** Does it matter what interval you use when you find the rate of change of a proportional relationship? Explain.

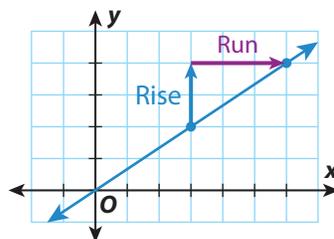
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# Calculating Slope

When the rate of change of a relationship is constant, every segment of its graph has the same steepness, and the segments together form a line. The constant rate of change is called the *slope* of the line.



The **slope** of a line is the ratio of the change in  $y$ -values (rise) for a segment of the graph to the corresponding change in  $x$ -values (run).



## EXAMPLE 2

FL CC 8.F.2.4

Find the slope of the line.

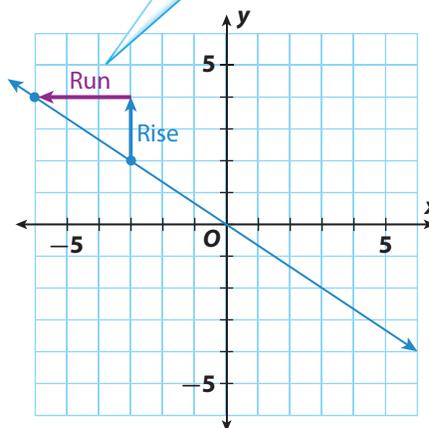
**STEP 1** Choose two points on the line.

**STEP 2** Find the change in  $y$ -values (rise) and the change in  $x$ -values (run) as you move from one point to the other.

rise = +2                      run = -3

**STEP 3** Slope =  $\frac{\text{rise}}{\text{run}}$   
 $= \frac{2}{-3}$   
 $= -\frac{2}{3}$

If you move up or right, the change is positive. If you move down or left, the change is negative.



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## YOUR TURN

4. The graph shows the rate at which water is leaking from a tank. The slope of the line gives the leaking rate in gallons per minute.

Rise = \_\_\_\_\_

Run = \_\_\_\_\_

Rate of leaking = \_\_\_\_\_ gallon(s) per minute

Leaking tank

